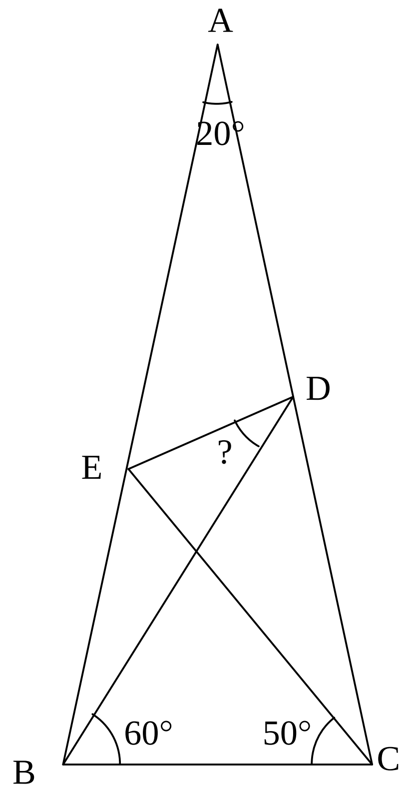
Langley's Adventitious Angle



ΔABC is an isosceles triangle where $\angle BAC = 20°$. Points D and E are on sides AC and AB so that $\angle BCE = 50°$ and $\angle CBD = 60°$. Find $\angle BDE$.

Solution:

ΔABC is isosceles $\Rightarrow \angle ABC = \angle ACB = 80°$.
Construct point F on AC so that ΔCBF is isosceles.
$\angle BFC = \angle BCF = 80° \Rightarrow \angle CBF = 20°$.
$\angle BEC = 180° – 80° – 50° = 50°$
$\Rightarrow \Delta BCE$ is isosceles
$\Rightarrow BE = BC$
$\Rightarrow BE = BF$ since ΔCBF is isosceles by construction.
$\Rightarrow \Delta BEF$ is isosceles
$\Rightarrow \Delta BEF$ is equilateral since $\angle EBF = 80° – 20° = 60°$
$\Rightarrow \angle BFE = 60°$.
$\Rightarrow \angle DFE = 180° – 80° – 60° = 40°$.

$\angle BDC = 180° – 80° – 60° = 40°$.
$\angle DBF = 60° – 20° = 40° = \angle BDC$
$\Rightarrow \Delta BDF$ is isosceles
$\Rightarrow BF = DF$
$\Rightarrow EF = BF = DF$
$\Rightarrow \Delta DEF$ is isosceles
$\Rightarrow \angle DEF = \dfrac{180° – 40°}{2} = 70°$
$\Rightarrow \angle BDE = 180° – 70° – 40° – 40° = 30°$.