SAVINGS AND LOANS PROBLEMS

Example 1: To find savings account balance
At the end of each month, you save $100 in a savings account. The bank pays 6% annual interest compounded monthly. How much will you have in your account at the end of 1 year?

Monthly interest = 0.06 ÷ 12 = 0.005

At the end of 1 month, balance = $100

At the end of 2 months, balance = $100 × 1.005 + 100

At the end of 3 months, balance = $(100 × 1.005 + 100) × 1.005 + 100

$= 100 \times 1.005^2 + 100 \times 1.005 + 100$

$$= 100\,\frac{1.005^3 - 1}{1.005 - 1}$$

At the end of 12 months, balance $= 100\,\dfrac{1.005^{12} - 1}{0.005}$

= $1233.56

In general, if you save $$M$ every month in an account that pays interest i compounded monthly, then the balance at the end of n months is $A_n = M\,\dfrac{(1+i)^n - 1}{i}$.

Example 2: To find monthly payment
A bank offers to pay 6% annual interest compounded monthly. How much do you need to save each month if you wish to have $10 000 in your savings account at the end of 5 years?

$$A_n = M\,\frac{(1+i)^n - 1}{i}$$

$$10\,000 = M\,\frac{1.005^{60} - 1}{0.005}$$

M = $143.33

Example 3: To find number of months
At the end of each month you save $100 in a savings account. The bank pays 6% annual interest compounded monthly. How long do you need to save if you wish to have $10 000?

$$A_n = M\,\frac{(1+i)^n - 1}{i}$$

$$10\,000 = 100\,\frac{1.005^n - 1}{0.005}$$

$1.005^n - 1 = 0.5$

$1.005^n = 1.5$

$n \log 1.005 = \log 1.5$

$n = 81.296$

$\therefore$ need to save for 82 months.

Example 4: To find loan balance

You took a $100 000 housing loan. The bank charges 6% annual interest compounded monthly. You pay back $1000 per month. How much do you owe the bank after 1 year?

Monthly interest = 0.06 ÷ 12 = 0.005

At the end of 1 month, loan balance $= 100\,000 \times 1.005 - 1000$

At the end of 2 months, loan balance $= (100\,000 \times 1.005 - 1000) \times 1.005 - 1000$

$= 100\,000 \times 1.005^2 - 1000 \times 1.005 - 1000$

At the end of 3 months, loan balance $= (100\,000 \times 1.005^2 - 1000 \times 1.005 - 1000) \times 1.005 - 1000$

$= 100\,000 \times 1.005^3 - 1000 \times 1.005^2 - 1000 \times 1.005 - 1000$

$$= 100\,000 \times 1.005^3 - 1000\,\frac{1.005^3 - 1}{1.005 - 1}$$

At the end of 12 months, loan balance $= 100\,000 \times 1.005^{12} - 1000\,\dfrac{1.005^{12} - 1}{0.005}$

= $93 832.22

In general, if you pay $M every month for an $L loan that charges interest i compounded monthly, then the loan balance at the end of n months is $A_n = L(1 + i)^n - M\,\dfrac{(1 + i)^n - 1}{i}$.

Example 5: To find monthly payment

You took a $100 000 housing loan to be repaid over 20 years. The bank charges 6% annual interest compounded monthly. How much do you need to pay every month?

Let $L(1 + i)^n - M\,\dfrac{(1 + i)^n - 1}{i} = 0$

$$M\,\frac{1.005^{240} - 1}{0.005} = 100\,000 \times 1.005^{240}$$

Monthly payment M = $716.43

Example 6: To find total loan amount

A bank offers you a 20–year housing loan that charges 6% annual interest compounded monthly. You can afford to pay $1000 per month. How much loan can you take?

Let $L(1 + i)^n - M\,\dfrac{(1 + i)^n - 1}{i} = 0$

$$L \times 1.005^{240} = 1000\,\frac{1.005^{240} - 1}{0.005}$$

Total loan amount L = $139 580.77

Example 7: To find number of months

You have taken a $100 000 housing loan from a bank. The bank charges 6% annual interest compounded monthly. You pay back $1000 per month. How long do you take to finish paying up the loan?

Let $L(1 + i)^n - M\,\dfrac{(1 + i)^n - 1}{i} = 0$

$100\,000 \times 1.005^n - 1000\,\dfrac{1.005^n - 1}{0.005} = 0$

$100\,000 \times 1.005^n = 200\,000$

$1.005^n = 2$

$n \log 1.005 = \log 2$

$n = 138.98$

It takes 139 months, i.e. 11 years and 7 months to pay up the loan.